

# $CP$ violation for $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$ in Perturbative QCD

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In the perturbative QCD (PQCD) approach we study the direct  $CP$  violation in  $B_c^+ \rightarrow D_{(s)}^+ \rho^0(\omega) \rightarrow D_{(s)}^+ \pi^+ \pi^-$  via the  $\rho - \omega$  mixing mechanism. We find that the  $CP$  violation can be enhanced by  $\rho - \omega$  mixing when the invariant masses of the  $\pi^+ \pi^-$  pairs are in the vicinity of the  $\omega$  resonance. For the decay process  $B_c^+ \rightarrow D^+ \rho^0(\omega) \rightarrow D^+ \pi^+ \pi^-$ , the maximum  $CP$  violation can reach 7.5 %.

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## I. INTRODUCTION

$CP$  violation is an important topic in particle physics. Within the Standard Model (SM),  $CP$  violation is originated from the weak phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix, along with the strong phase which usually arises from strong interactions [1, 2]. In the past few years more attention has been focused on the decays of  $B$  meson system both theoretically and experimentally. Some events of  $B_c$  mesons have been observed at Tevatron. Fortunately, a great number of events will appear at LHC in the foreseeable future. Recently, the LHCb Collaboration focused on three-body decays channels of  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  and  $B^\pm \rightarrow K^\pm \pi^+ \pi^-$  to probe large  $CP$  violation [3–5]. The intriguing discoveries present us opportunities to detect  $CP$  violation mechanism. The research of three-body decays of  $B_c$  meson may be next topic for the LHCb experiments in the following years. In this paper, we focus on the interference from intermediate  $\rho$  and  $\omega$  mesons in the  $B_c$  meson decays.

There has been remarkable progress in the study of exclusive  $\overline{B}_d^0 \rightarrow h_1 h_2$  and  $B^\pm \rightarrow h_1 h_2$  decays, where  $h_1, h_2$  are light pseudo-scalar and/or vector mesons. Historically, these decays were calculated in the so-called naive factorization approach [6], which was improved by including perturbative QCD contributions [7, 8]. Currently, there are three popular theoretical approaches to study the dynamics of these decays, which go under the name QCD factorization (QCDF) [9], perturbative QCD (PQCD) [10, 11], and soft-collinear effective theory (SCET) [12]. All three are based on power expansion in  $1/m_b$ , where  $m_b$  is the  $b$ -quark mass. Factorization of the hadronic matrix elements  $\langle h_1 h_2 | \mathcal{O}_i | B \rangle$ , where  $\mathcal{O}_i$  is typically a four-quark or a magnetic moment type operator, is shown to exist in the leading power in  $1/m_b$  in a class of decays. But, these methods are different significantly

due to the collinear degree or transverse momenta. The power counting is different from the hard kernels between QCDF and PQCD. It is important to extract the strong phase difference for  $CP$  violation. The more different feature of QCDF and PQCD is the strong interaction scale at which of PQCD is low, typically of order  $1 \sim 2$  GeV, the case of QCDF is order  $O(m_b)$  for the Wilson coefficients.

Direct  $CP$  violating asymmetries in  $b$ -hadron decays occur through the interference of at least two amplitudes with the weak phase difference  $\phi$  and the strong phase difference  $\delta$ . The weak phase difference is determined by the CKM matrix, while the strong phase is usually difficult to control. In order to acquire a large  $CP$  violating asymmetries signal, we need to apply some phenomenological mechanism to obtain a large  $\delta$ . It has been shown that the charge symmetry violating mixing between  $\rho$  and  $\omega$  can be used to obtain a large strong phase difference which is required for large  $CP$  violating asymmetries [13–21]. In this paper, we will investigate the  $CP$  violation via  $\rho - \omega$  mixing using PQCD approach in the decays of  $B_c$  mesons.

In the perturbative QCD approach, at the rest frame of heavy  $B$  meson,  $B$  meson decays into two heavy quarks with large momenta. The hard interaction dominates the decay amplitude from short distance due to not enough time to exchange soft gluons with final mesons. Since the final mesons move very fast, a hard gluon kicks the light spectator quark of  $B$  meson to form a fast moving final meson. Hence, the hard interaction is consist of six quark operator. The non-perturbative dynamics are included in the meson wave function which can be extracted from experiment. The hard one can be calculated by perturbation theory. Since  $B_c$  meson has two heavy quarks ( $b$  and  $c$  quark), each of them can decay individually. It has been pointed out the  $c$ -quark decay processes can only produce about 1%  $CP$  violation [17]. Hence, we only consider the contribution of  $b$ -quark decay in the processes that we are considering.

The remainder of this paper is organized as follows. In Sec. II we present the form of the effective Hamiltonian. In Sec. III we give the calculating formalism of  $CP$  violation from  $\rho - \omega$  mixing in  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$  decay.

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We present the numerical results in Sec. IV. A summary and discussion are included in the Sec. V. The related functions defined in the text are given in Appendix.

## II. THE EFFECTIVE HAMILTONIAN

Based on the operator product expansion, the effective weak Hamiltonian can be expressed as [22]:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q=u,c} \xi_q [(C_1(\mu) O_1^q(\mu)) + C_2(\mu) O_2^q(\mu)] + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\}, \quad (1)$$

where  $\xi_q = V_{qp} V_{qb}^*$  ( $p = d, s$ ) are CKM matrix elements, and  $c_i(\mu)$  ( $i = 1, 2, \dots, 10$ ) are the Wilson coefficients, which are calculable in the renormalization group improved perturbation theory and are scale dependent. In the present case, we work with the renormalization scheme independent Wilson coefficients and use the values of the Wilson coefficients at the renormalization scale  $\mu \approx m_b$ .  $G_F$  represents Fermi constant and  $O_i$  is the effective four quark operator, which can be written as

$$\begin{aligned} O_1^u &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\ O_2^u &= \bar{d}_\mu \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) b, \\ O_3 &= \bar{d}_\mu \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_4 &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\ O_5 &= \bar{d}_\mu \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_6 &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_7 &= \frac{3}{2} \bar{d}_\mu \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_8 &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_9 &= \frac{3}{2} \bar{d}_\mu \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_{10} &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \end{aligned} \quad (2)$$

where  $\alpha$  and  $\beta$  are color indices, and  $q'$  running through all the light flavour quarks. In Eq.(2)  $O_1^u$  and  $O_2^u$  are the tree level and QCD corrected operators,  $O_3$ – $O_6$  are QCD penguin operators and  $O_7$ – $O_{10}$  are the operators associated with electroweak penguin diagrams.

In the PQCD approach, three scales are involved: the W-boson mass  $m_W$  associated weak interaction, the hard

scale  $t$ , and factorization scale  $1/b$  ( $b$  is the conjugate variable of the parton transverse momenta  $k_T$ ). The decay amplitude is then factorized into the convolution of the hard subamplitude, the Wilson coefficient and the Sudakov factor with the meson wave functions, all of which are well-defined and gauge invariant. Therefore, the three scale factorization formula for exclusive nonleptonic  $B$  meson decays is then written as

$$C(t) \otimes H(x, t) \otimes \Phi(x) \otimes \exp[-s(P, b) - 2 \int_{1/b}^t \frac{d\mu}{\mu} \gamma_q(\alpha_s(\mu))], \quad (3)$$

where  $C(t)$  are the corresponding Wilson coefficients.  $\Phi(x)$  are the meson wave functions and the variable  $t$  denotes the largest mass scale of hard process  $H$  that is six-quark effective theory. Sudakov factor coming from renormalization summation and threshold summation is introduced to solve the endpoint diverging. It can handle with endpoint diverging problem properly by introducing Sudakov factor [23]. The Sudakov evolution  $\exp[-s(P, b)]$  are from the resummation of double logarithms  $\ln^2(Pb)$ , with  $P$  denoting the dominant light-cone component of meson momentum.  $\gamma_q = -\alpha_s/\pi$  is quark anomalous dimension in axial gauge.

## III. CP VIOLATION IN $B_c^+ \rightarrow D_{(s)}^+ \rho^0(\omega) \rightarrow D_{(s)}^+ \pi^+ \pi^-$

### A. Formalism

In the vector meson dominance model, the photon propagator is dressed by coupling to vector meson. Based on the same mechanism,  $\rho - \omega$  mixing was proposed. According to the effective Hamiltonian, the amplitude  $\mathcal{A}$  for  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$  can be divided into two parts:

$$\mathcal{A} = \langle D_{(s)}^+ \pi^+ \pi^- | H^T | B_c^+ \rangle + \langle D_{(s)}^+ \pi^+ \pi^- | H^P | B_c^+ \rangle, \quad (4)$$

with  $H^T$  and  $H^P$  being the Hamiltonian for the tree and penguin operators, respectively.

We can define the relative magnitudes and phases between the tree and penguin operator contributions as follows:

$$\mathcal{A} = \langle D_{(s)}^+ \pi^+ \pi^- | H^T | B_c^+ \rangle [1 + r e^{i(\delta+\phi)}], \quad (5)$$

where  $\delta$  and  $\phi$  are strong and weak phases, respectively.  $\phi$  arises from the CP-violating phase in the CKM matrix, which is  $\arg[V_{tb} V_{tq}^* / (V_{ub} V_{uq}^*)]$  ( $q = d, s$ ). The parameter  $r$  is the absolute value of the ratio of penguin and tree amplitudes:

$$r \equiv \left| \frac{\langle D_{(s)}^+ \pi^+ \pi^- | H^P | B_c^+ \rangle}{\langle D_{(s)}^+ \pi^+ \pi^- | H^T | B_c^+ \rangle} \right|. \quad (6)$$

The amplitude for  $B_c^- \rightarrow D_{(s)}^- \pi^+ \pi^-$  is

$$\bar{A} = \langle D_{(s)}^- \pi^+ \pi^- | H^T | B_c^- \rangle + \langle D_{(s)}^- \pi^+ \pi^- | H^P | B_c^- \rangle. \quad (7)$$

In this work, we only consider  $\rho^0$  and  $\omega$  resonances. The  $CP$  violating asymmetry for  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$  is defined as

$$A_{cp} \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2}. \quad (8)$$

From Equation (8), one can find the  $CP$  violation depends on the weak phase difference and the strong phase difference. The weak phase is determined for a specific decay process. Hence, in order to obtain a large  $CP$  violation, we need some mechanism to make  $\sin \delta$  large. It has been found that  $\rho - \omega$  mixing (which was proposed based on vector meson dominance [24]) leads to a large strong phase difference [14–21]. Based on  $\rho - \omega$  mixing and working to the first order of isospin violation, we have the following results:

$$\langle D_{(s)}^+ \pi^+ \pi^- | H^T | B_c^+ \rangle = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} t_\omega + \frac{g_\rho}{s_\rho} t_\rho, \quad (9)$$

$$\langle D_{(s)}^+ \pi^+ \pi^- | H^P | B_c^+ \rangle = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} p_\omega + \frac{g_\rho}{s_\rho} p_\rho. \quad (10)$$

where  $t_v$  ( $v = \rho$  or  $\omega$ ) is the tree amplitudes and  $p_v$  is the penguin amplitudes for producing an intermediate vector meson  $V$ .  $g_\rho$  is the coupling for  $\rho^0 \rightarrow \pi^+ \pi^-$ ;  $\tilde{\Pi}_{\rho\omega}$  is the effective  $\rho - \omega$  mixing amplitude which also effectively includes the direct coupling  $\omega \rightarrow \pi^+ \pi^-$ .  $s_V$ ,  $m_V$  and  $\Gamma_V$  ( $V = \rho$  or  $\omega$ ) is the inverse propagator, mass and decay rate of the vector meson  $V$ , respectively.

$$s_V = s - m_V^2 + im_V \Gamma_V. \quad (11)$$

with  $\sqrt{s}$  being the invariant masses of the  $\pi^+ \pi^-$  pairs.

We stress that the direct coupling  $\omega \rightarrow \pi^+ \pi^-$  is effectively absorbed into  $\tilde{\Pi}_{\rho\omega}$ , leading to the explicit  $s$  dependence of  $\tilde{\Pi}_{\rho\omega}$  [25]. However, the  $s$  dependence of  $\tilde{\Pi}_{\rho\omega}$  is negligible in practice. We can make the expansion  $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2) \tilde{\Pi}'_{\rho\omega}(m_\omega^2)$ . The  $\rho - \omega$  mixing parameters were determined in the fit of Gardner and O'Connell [26]:

$$\begin{aligned} \Re \tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -3500 \pm 300 \text{ MeV}^2, \\ \Im \tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -300 \pm 300 \text{ MeV}^2, \\ \tilde{\Pi}'_{\rho\omega}(m_\omega^2) &= 0.03 \pm 0.04. \end{aligned} \quad (12)$$

From Eqs. (4)(5)(9)(10) one has

$$r e^{i\delta} e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega} p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega} t_\omega + s_\omega t_\rho}, \quad (13)$$

Defining

$$\frac{p_\omega}{t_\rho} \equiv r' e^{i(\delta_q + \phi)}, \quad \frac{t_\omega}{t_\rho} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_\rho}{p_\omega} \equiv \beta e^{i\delta_\beta}, \quad (14)$$

where  $\delta_\alpha$ ,  $\delta_\beta$  and  $\delta_q$  are strong phases. One finds the following expression from Eqs. (13)(14):

$$r e^{i\delta} = r' e^{i\delta_q} \frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{\tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha} + s_\omega}. \quad (15)$$

In order to get the  $CP$  violating asymmetry in Eq. (8),  $\sin \phi$  and  $\cos \phi$  are needed. The weak phase  $\phi$  is fixed by the CKM matrix elements. In the Wolfenstein parametrization [27], one has

$$\begin{aligned} \sin \phi &= \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}, \\ \cos \phi &= \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}. \end{aligned} \quad (16)$$

## B. Computational details

From Equations (8)(13)(14), in order to obtain the formulas of the  $CP$  violation, we calculate the amplitudes  $t_\rho$ ,  $t_\omega$ ,  $p_\rho$  and  $p_\omega$  in PQCD approach, which can be decomposed in terms of tree-level and penguin-level amplitudes due to the CKM matrix elements of  $V_{ud}V_{ub}^*$ ,  $V_{us}V_{ub}^*$ ,  $V_{td}V_{tb}^*$  and  $V_{ts}V_{tb}^*$ . In the following, we calculate the decay amplitudes for  $B_c^+ \rightarrow D^+ \rho^0(\omega)$  and  $B_c^+ \rightarrow D_s^+ \rho^0(\omega)$  which we will use in the next paragraph. The PQCD function of  $F$  and  $M$  can be found in appendix.

### 1. The decay amplitudes of $B_c^+ \rightarrow D^+ \rho^0(\omega)$

With the Hamiltonian (1), depending on CKM matrix elements of  $V_{ud}V_{ub}^*$ ,  $V_{us}V_{ub}^*$ ,  $V_{td}V_{tb}^*$  and  $V_{ts}V_{tb}^*$ , the decay amplitudes for  $B_c^+ \rightarrow D^+ \rho^0$  in PQCD can be written as

$$\sqrt{2} \mathcal{A}(B_c^+ \rightarrow D^+ \rho^0) = V_{ud}V_{ub}^* t_\rho + V_{td}V_{tb}^* p_\rho \quad (17)$$

where

$$\begin{aligned} t_\rho &= (C_1 + \frac{1}{3}C_2)F_e^{LL} + C_2M_e^{LL} \\ &\quad + (C_2 + \frac{1}{3}C_1)F_a^{LL} + C_1M_a^{LL} \end{aligned} \quad (18)$$

and

$$\begin{aligned} P_\rho &= (C_2 + \frac{1}{3}C_1)F_a^{LL} + C_1M_a^{LL} \\ &\quad - [(\frac{3}{2}C_{10} - C_3 + \frac{1}{2}C_9)M_e^{LL} \\ &\quad - (C_3 + C_9)M_a^{LL} + (-C_5 + \frac{1}{2}C_7)M_e^{LR} \\ &\quad + (-C_4 - \frac{1}{3}C_3 - C_{10} - \frac{1}{3}C_9)F_a^{LL} \\ &\quad + (C_{10} + \frac{5}{3}C_9 - \frac{1}{3}C_3 - C_4 - \frac{3}{2}C_7 - \frac{1}{2}C_8)F_e^{LL} \\ &\quad + (-C_6 - \frac{1}{3}C_5 + \frac{1}{2}C_8 + \frac{1}{6}C_7)F_e^{SP} \\ &\quad - (C_5 + C_7)M_a^{LR} \\ &\quad + (-C_6 - \frac{1}{3}C_5 - C_8 - \frac{1}{3}C_7)F_a^{SP}] \end{aligned} \quad (19)$$

The decay amplitude for  $B_c^+ \rightarrow D^+\omega$  can be written as

$$\sqrt{2}\mathcal{A}(B_c^+ \rightarrow D^+\omega) = V_{ud}V_{ub}^*t_\omega - V_{td}V_{tb}^*p_\omega, \quad (20)$$

where

$$t_\omega = (C_1 + \frac{1}{3}C_2)F_e^{LL} + C_2M_e^{LL} - [(C_2 + \frac{1}{3}C_1)F_a^{LL} + C_1M_a^{LL}] \quad (21)$$

and

$$\begin{aligned} p_\omega = & (C_2 + \frac{1}{3}C_1)F_a^{LL} + C_1M_a^{LL} \\ & + [(2C_4 + C_3 + \frac{1}{2}C_{10} - \frac{1}{2}C_9)M_e^{LL} \\ & + (C_3 + C_9)M_e^{LL} \\ & + (C_5 - \frac{1}{2}C_7)M_e^{LR} + (C_5 + C_7)M_a^{LR} \\ & + (C_4 + \frac{1}{3}C_3 + C_{10} + \frac{1}{3}C_9)F_a^{LL} \\ & + (\frac{7}{3}C_3 + \frac{5}{3}C_4 + \frac{1}{3}(C_9 - C_{10}))F_e^{LL} \\ & + (2C_5 + \frac{2}{3}C_6 + \frac{1}{2}C_7 + \frac{1}{6}C_8)F_e^{LR} \\ & + (C_6 + \frac{1}{3}C_5 - \frac{1}{2}C_8 - \frac{1}{6}C_7)F_e^{SP} \\ & + (C_6 + \frac{1}{3}C_5 + C_8 + \frac{1}{3}C_7)F_a^{SP}] \end{aligned} \quad (22)$$

Based on the definition of (14), we can get

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega}{t_\rho}, \quad (23)$$

$$\beta e^{i\delta_\beta} = \frac{p_\rho}{p_\omega}, \quad (24)$$

$$r' e^{i\delta_q} = \frac{p_\omega}{t_\rho} \times \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right|, \quad (25)$$

where

$$\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \frac{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}{(1 - \lambda^2/2)(\rho^2 + \eta^2)} \quad (26)$$

## 2. The decay amplitudes of $B_c^+ \rightarrow D_s^+\rho^0(\omega)$

The decay amplitudes for  $B_c^+ \rightarrow D_s^+\rho^0$  can be written as

$$\sqrt{2}\mathcal{A}(B_c^+ \rightarrow D_s^+\rho^0) = V_{us}V_{ub}^*t_\rho - V_{ts}V_{tb}^*p_\rho, \quad (27)$$

where

$$t_\rho = (C_1 + \frac{1}{3}C_2)F_e^{LL} + C_2M_e^{LL} \quad (28)$$

and

$$\begin{aligned} p_\rho = & [(\frac{1}{2}(3C_9 + C_{10})F_e^{LL} + \frac{1}{2}(3C_7 + C_8))F_e^{LR} \\ & + \frac{3}{2}C_{10}M_e^{LL} + \frac{3}{2}C_8M_e^{SP}] \end{aligned} \quad (29)$$

The decay amplitudes for  $B_c^+ \rightarrow D_s^+\omega$  can be written as

$$\sqrt{2}\mathcal{A}(B_c^+ \rightarrow D_s^+\omega) = V_{us}V_{ub}^*t_\omega - V_{ts}V_{tb}^*p_\omega, \quad (30)$$

where

$$t_\omega = (C_1 + \frac{1}{3}C_2)F_e^{LL} + C_2M_e^{LL} \quad (31)$$

and

$$\begin{aligned} p_\omega = & [(2C_4 + \frac{1}{2}C_{10})M_e^{LL} + (2C_6 + \frac{1}{2}C_8)M_e^{SP} \\ & + (2C_3 + \frac{2}{3}C_4 + \frac{1}{2}C_9 + \frac{1}{6}C_{10})F_e^{LL} \\ & + (2C_5 + \frac{2}{3}C_6 + \frac{1}{2}C_7 + \frac{1}{6}C_8)F_e^{LR}] \end{aligned} \quad (32)$$

Similarity, we can also obtain the strong phase from the Eqs. (23)(24)(25)(26).

## IV. NUMERICAL RESULTS

### A. Input parameters

In the numerical calculations, we have several parameters. The Wilson coefficients,  $C_i(\mu)$ , take the following values [11]:

$$\begin{aligned} C_1 &= -0.2703, & C_2 &= 1.1188, \\ C_3 &= 0.0126, & C_4 &= -0.0270, \\ C_5 &= 0.0085, & C_6 &= -0.0326, \\ C_7 &= 0.0011, & C_8 &= 0.0004, \\ C_9 &= -0.0090, & C_{10} &= 0.0022, \end{aligned} \quad (33)$$

The CKM matrix, which should be determined from experiments, can be expressed in terms of the Wolfenstein parameters,  $A, \lambda, \rho$  and  $\eta$  [27]:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (34)$$

where  $O(\lambda^4)$  corrections are neglected. The latest values for the parameters in the CKM matrix are [28]:

$$\begin{aligned} \lambda &= 0.2253 \pm 0.0007, & A &= 0.808_{-0.015}^{+0.022}, \\ \bar{\rho} &= 0.132_{-0.014}^{+0.022}, & \bar{\eta} &= 0.341 \pm 0.013, \end{aligned} \quad (35)$$

with

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \quad (36)$$

From Eqs. (35) (36) we have

$$0.121 < \rho < 0.158, \quad 0.336 < \eta < 0.363. \quad (37)$$

We adopt the results from [28]:

$$\begin{aligned}
|V_{ub}| &= (3.89 \pm 0.44) \times 10^{-3}, & |V_{ud}| &= 0.97425, \\
|V_{cb}| &= 0.0406, & |V_{cd}| &= 0.23 \\
|V_{us}| &= 0.2252, & |V_{cs}| &= 1.023, \\
m_b &= 4.2 \text{ GeV}, & m_0^\pi &= 1.4 \text{ GeV}, \\
m_0^K &= 1.6 \text{ GeV}, & m_0^{\eta_q} &= 1.07 \text{ GeV}, \\
\gamma &= (73_{-25}^{+22})^\circ, & m_c &= 1.27 \text{ GeV}, \\
m_0^{\eta_s} &= 1.92 \text{ GeV}, & \Lambda_{QCD}^5 &= 0.112 \text{ GeV}.
\end{aligned} \tag{38}$$

### B. $CP$ violation in $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$

In the numerical results, we find that the  $CP$  violation can be enhanced via  $\rho - \omega$  mixing for the decay channel  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$  when the invariant mass of  $\pi^+ \pi^-$  is in the vicinity of the  $\omega$  resonance. The  $CP$  violation depends on the weak phase difference from CKM matrix elements and the strong phase difference which is difficult to control. The CKM matrix elements, which relate to  $\rho$ ,  $\eta$ ,  $\lambda$  and  $A$ , are given in Eq.(35). The uncertainties due to the CKM matrix elements come from  $\rho$ ,  $\eta$ ,  $\lambda$  and  $A$ . In our numerical calculations, we let  $\rho$ ,  $\eta$ ,  $\lambda$  and  $A$  vary among the limiting values. The numerical results are shown from Fig.1 to Fig.6 with the different parameter values of CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$ , respectively. In Fig.1 and Fig.2, we give the central value of  $CP$  violating asymmetry as a function of  $\sqrt{s}$ . From the Fig.1 and Fig.2 one can see the  $CP$  violation parameter is dependent on  $\sqrt{s}$  and changes rapidly due to  $\rho - \omega$  mixing when the invariant mass of  $\pi^+ \pi^-$  is in the vicinity of the  $\omega$  resonance. We can see that the  $CP$  violating asymmetry vary from around 2% to around 7.5% for the decay channel of  $B_c \rightarrow D^+ \pi^+ \pi^-$  when  $\sqrt{s} = 0.786 \text{ GeV}$  in Fig.1. As can be seen from Fig.2 the  $CP$  violating asymmetry vary from around 5.3% to around 7.2% for the decay channel of  $B_c \rightarrow D_s^+ \pi^+ \pi^-$  when  $\sqrt{s} = 0.774 \text{ GeV}$ .

From Eq.(8), we can see that the  $CP$  violating parameter is related to  $\sin\delta$  and  $r$ . The plots of  $\sin\delta$  and  $r$  as a function of  $\sqrt{s}$  are shown in Fig.3, Fig.4, Fig.5 and Fig.6, respectively. It can be seen that  $\sin\delta$  and  $r$  change sharply at the range of  $\rho - \omega$  resonance. In Fig.3 and Fig.4, we show the plot of  $\sin\delta$  as a function of  $\sqrt{s}$ . We can see that the  $\rho - \omega$  mixing mechanism leads to the strong phase at the  $\omega$  resonance for the processes of  $B_c^+ \rightarrow D^+ \pi^+ \pi^-$  and  $B_c^+ \rightarrow D_s^+ \pi^+ \pi^-$ . One can find  $\rho - \omega$  mixing make the  $\sin\delta$  value oscillate from  $-0.03$  to  $-0.095$  and  $0.118$  to  $0.223$  for the decay processes of  $B_c^+ \rightarrow D^+ \pi^+ \pi^-$  and  $B_c^+ \rightarrow D_s^+ \pi^+ \pi^-$ , respectively. From Fig.5 and Fig.6, one can see that  $r$  increases slowly for the channel of  $B_c^+ \rightarrow D_{(s)}^+ \pi^+ \pi^-$  when the invariant masses of the  $\pi^+ \pi^-$  pairs are in the vicinity of

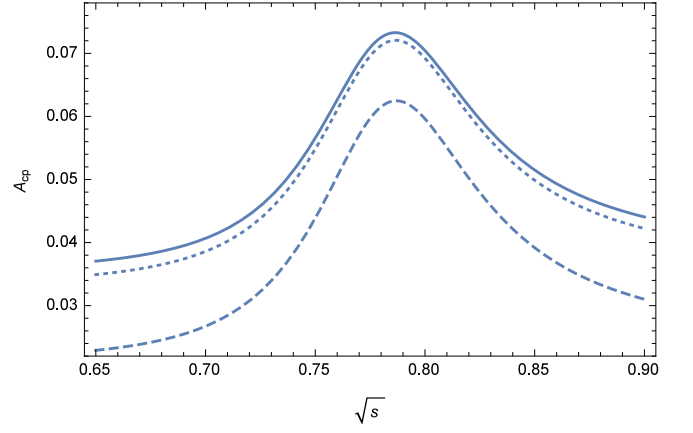


FIG. 1. The  $CP$  violating asymmetry,  $A_{cp}$ , as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D^+ \pi^+ \pi^-$ , respectively.

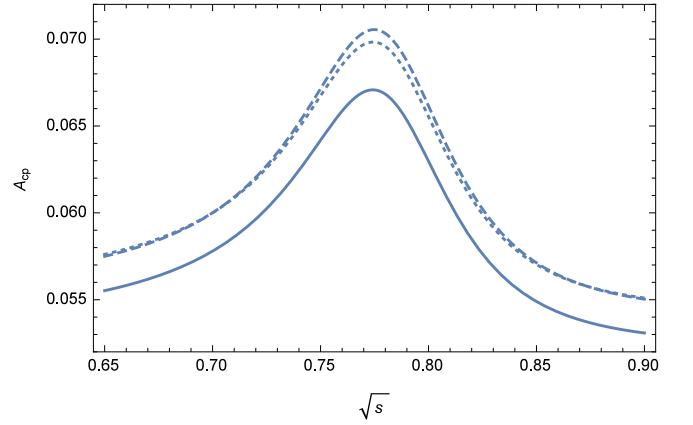


FIG. 2. The  $CP$  violating asymmetry,  $A_{cp}$ , as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D_s^+ \pi^+ \pi^-$ , respectively.

the  $\omega$  resonance.

For the processes of above decay channels,  $\rho - \omega$  mixing does enhance  $CP$  violating asymmetries and provide a mechanism for producing large  $CP$  violation in perturbative QCD. Meanwhile we find  $\rho - \omega$  mixing presents strong phase so as to make  $\sin\delta$  big and can also change the value of  $r$ . However, we find that the effect of the change of  $r$  on  $A_{cp}$  is small compared with the case of  $\sin\delta$  for the processes we are considering.

## V. SUMMARY AND DISCUSSION

In this paper, we have studied the  $CP$  violation in the decay of  $B_c^+ \rightarrow D_{(s)}^+ \rho^0(\omega) \rightarrow D_{(s)}^+ \pi^+ \pi^-$  due to the contribution of  $\rho - \omega$  mixing in PQCD approach. It is

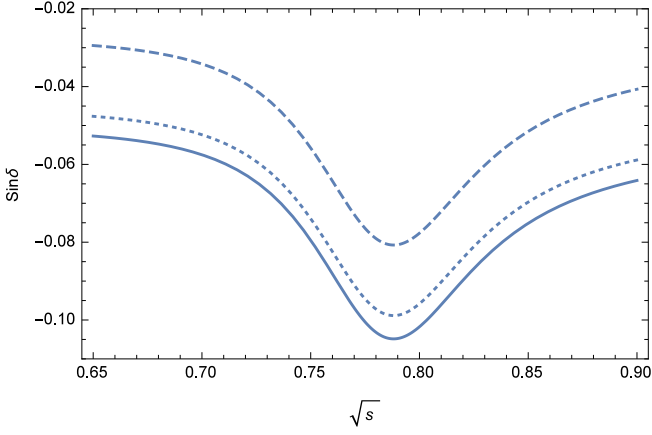


FIG. 3.  $\sin\delta$  as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D^+\pi^+\pi^-$ , respectively.

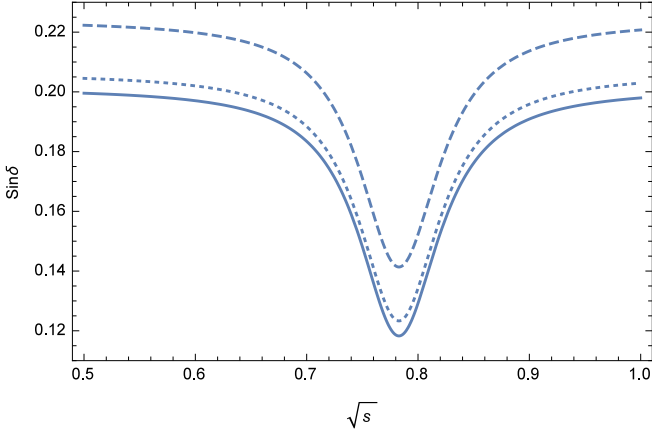


FIG. 4.  $\sin\delta$  as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D_s^+\pi^+\pi^-$ , respectively.

found that  $\rho - \omega$  mixing can cause a large strong phase difference so that large  $CP$  violation can be obtained at the  $\omega$  resonance. As a result, it is found that the maximum  $CP$  violation can reach 7.5%.

The LHC is a proton-proton collider currently have started at CERN. With the designed center-of-mass energy 14 TeV and luminosity  $L = 10^{34} \text{cm}^{-2} \text{s}^{-1}$ , the LHC gives access to high energy frontier at TeV scale and an opportunity to further improve the consistency test for the CKM matrix. The production rates for heavy quark flavours will be large at the LHC, and the  $b\bar{b}$  production cross section will be of the order 0.5 mb, providing as many as  $0.5 \times 10^{12}$  bottom events per year [29]. The heavy quark physics is one of the main topics of LHC experiments. Especially, LHCb detector is designed to make precise studies on  $CP$  asymmetries and rare decays of b-hadron systems. The other two experiments, ATLAS

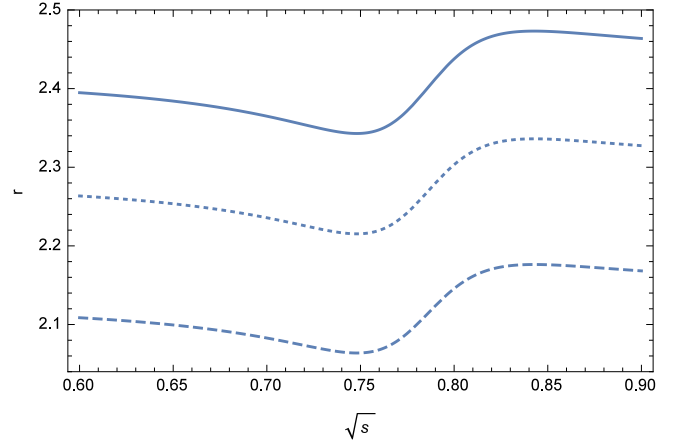


FIG. 5.  $r$  as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D^+\pi^+\pi^-$ , respectively.

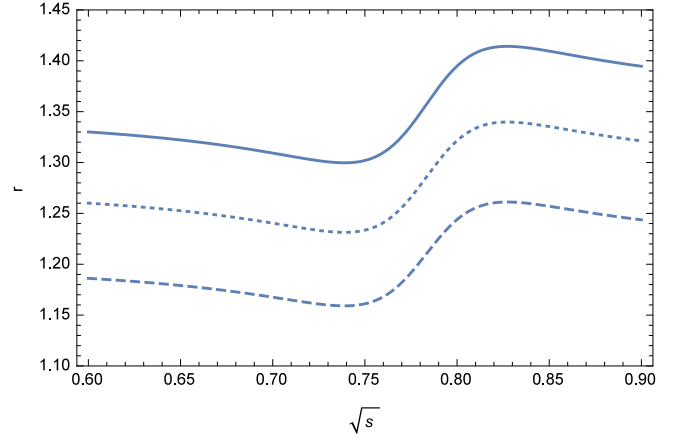


FIG. 6.  $r$  as a function of  $\sqrt{s}$  for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of  $B_c^+ \rightarrow D_s^+\pi^+\pi^-$ , respectively.

and CMS, are optimized for discovering new physics and will complete most of their  $B$  physics program within the first few years [29, 30]. Recently, the LHCb collaboration found clear evidence for direct  $CP$  violation in some three-body decay channels in charmless decays of  $B$  meson. Meanwhile, large  $CP$  violation is obtained in  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  in the region  $0.6 \text{ GeV}^2 < m_{\pi^+ \pi^-}^2 < 0.8 \text{ GeV}^2$  and  $m_{\pi^+ \pi^-}^2 > 14 \text{ GeV}^2$ . A zoom of the low  $\pi^+ \pi^-$  invariant mass from the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay, showing the region  $0.6 \text{ GeV}^2 < m_{\pi^+ \pi^-}^2 < 0.8 \text{ GeV}^2$  zone[31]. Fortunately, the experiments on  $B_c$  mesons have been planned at LHCb. The predicted  $CP$  violation for the decay processes we are considering can be searched in the region of the invariant masses of  $\pi^+ \pi^-$  associated  $\omega$  resonance on these experiments.

## ACKNOWLEDGMENTS

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## VI. APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

In this paper, the related functions can be written as [10, 11][32][33]:

$$\begin{aligned} \mathcal{F}_e^{LL} = & 2\sqrt{\frac{2}{3}}C_F f_B f_P \pi M_B^4 \int_0^1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \\ & \times \phi_D(x_2, b_2) \{[(1-2r_D)x_2 + (r_D-2)r_b] \\ & \times \alpha_s(t_a) h_e(\alpha_e, \beta_a, b_1, b_2) S_t(x_2) \exp[-S_{ab}(t_a)] \\ & - (r_D-2)r_D(x_1-1)\alpha_s(t_b) h_e(\alpha_e, \beta_b, b_2, b_1) \\ & \times S_t(x_1) \exp[-S_{ab}(t_b)]\}, \end{aligned} \quad (39)$$

$$\mathcal{F}_e^{LR} = -\mathcal{F}_e^{LL} \quad (40)$$

$$\begin{aligned} \mathcal{F}_e^{SP} = & -4\sqrt{\frac{2}{3}}C_F f_B f_P \pi M_B^4 \int_0^1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \\ & \times \phi_D(x_2, b_2) \{[r_D(4r_b - x_2 - 1) - r_b + 2]\alpha_s(t_a) \\ & \times h_e(\alpha_e, \beta_a, b_1, b_2) S_t(x_2) \exp[-S_{ab}(t_a)] \\ & + [r_D(2-4x_1) + x_1]\alpha_s(t_b) h_e(\alpha_e, \beta_b, b_2, b_1) \\ & \times S_t(x_1) \exp[-S_{ab}(t_b)]\}. \end{aligned} \quad (41)$$

$$\begin{aligned} \mathcal{M}_e^{LL} = & \frac{8}{3}C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_2 db_3 \\ & \times \phi_D(x_2, b_2) \phi_P^A(x_3) \{[r_D(1-x_1-x_2) + x_1 \\ & + x_3 - 1]\alpha_s(t_c) h_e(\beta_c, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_c)] \\ & - [r_D(1-x_1-x_2) + 2x_1 + x_2 - x_3 - 1]\alpha_s(t_d) \\ & \times h_e(\beta_d, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_d)]\}, \end{aligned} \quad (42)$$

$$\begin{aligned} \mathcal{M}_e^{LR} = & \frac{8}{3}C_F f_B \pi M_B^4 r_P(1+r_D) \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_2 db_3 \\ & \times \phi_D(x_2, b_2) \{[(x_1+x_3-1+r_D(2x_1+x_2+x_3-2)) \\ & \times \phi_P^P(x_3) + (x_1+x_3-1+r_D(x_3-x_2))\phi_P^T(x_3)] \\ & \times \alpha_s(t_c) h_e(\beta_c, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_c)] \\ & - [(x_1-x_3+r_D(2x_1+x_2-x_3-1))\phi_P^P(x_3) \\ & + (x_3-x_1+r_D(x_3+x_2-1))\phi_P^T(x_3)]\alpha_s(t_d) \\ & \times h_e(\beta_d, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_d)]\}, \end{aligned} \quad (43)$$

$$\begin{aligned} \mathcal{M}_e^{SP} = & \frac{8}{3}C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_2 db_3 \\ & \times \phi_D(x_2, b_2) \phi_P^A(x_3) \{[r_D(x_1+x_2-1) - 2x_1 - x_2 \\ & - x_3 + 2]\alpha_s(t_c) h_e(\beta_c, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_c)] \\ & - [x_3 - x_1 - r_D(1-x_1-x_2)]\alpha_s(t_d) \\ & \times h_e(\beta_d, \alpha_e, b_3, b_2) \exp[-S_{cd}(t_d)]\}, \end{aligned} \quad (44)$$

$$\begin{aligned} \mathcal{F}_a^{LL} = \mathcal{F}_a^{LR} = & -8C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_2 db_3 \\ & \times \phi_D(x_2, b_2) \{[\phi_P^A(x_3)(x_3-2r_D r_c) + r_P[\phi_P^P(x_3) \\ & \times (2r_D(x_3+1) - r_c) + \phi_P^T(x_3)(r_c+2r_D(x_3-1))]] \\ & \times \alpha_s(t_e) h_e(\alpha_a, \beta_e, b_2, b_3) \exp[-S_{ef}(t_e)] S_t(x_3) \\ & - [x_2 \phi_P^A(x_3) + 2r_P r_D(x_2+1)\phi_P^P(x_3)]\alpha_s(t_f) \\ & \times h_e(\alpha_a, \beta_f, b_3, b_2) \exp[-S_{ef}(t_f)] S_t(x_2)\}, \end{aligned} \quad (45)$$

$$\begin{aligned} \mathcal{F}_a^{SP} = & 16C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_2 db_3 \\ & \times \phi_D(x_2, b_2) \{[-\phi_P^A(x_3)(2r_D - r_c) + r_P[\phi_P^P(x_3) \\ & \times (4r_c r_D - x_3) + \phi_P^T(x_3)x_3]]\alpha_s(t_e) h_e(\alpha_a, \beta_e, b_2, b_3) \\ & \times \exp[-S_{ef}(t_e)] S_t(x_3) - [x_2 r_D \phi_P^A(x_3) + 2r_P \phi_P^P(x_3)] \\ & \times \alpha_s(t_f) h_e(\alpha_a, \beta_f, b_3, b_2) \exp[-S_{ef}(t_f)] S_t(x_2)\}; \end{aligned} \quad (46)$$

$$\begin{aligned} \mathcal{M}_a^{LL} = & -\frac{8}{3}C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_2 db_1 db_2 \\ & \times \phi_D(x_2, b_2) \{[\phi_P^A(x_3)(r_c - x_1 + x_2) + r_P r_D \\ & \times [\phi_P^T(x_3)(x_2 - x_3) + \phi_P^P(x_3)(4r_c - 2x_1 + x_2 \\ & + x_3)]]\alpha_s(t_g) h_e(\beta_g, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_g)] \\ & + [-\phi_P^A(x_3)(r_b + x_1 + x_3 - 1) + r_P r_D[(x_2 - x_3) \\ & \times \phi_P^T(x_3) - \phi_P^P(x_3)(4r_b + 2x_1 + x_2 + x_3 - 2)]] \\ & \times \alpha_s(t_h) h_e(\beta_h, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_h)]\}, \end{aligned} \quad (47)$$

$$\begin{aligned} \mathcal{M}_a^{LR} = & \frac{8}{3}C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_2 db_1 db_2 \\ & \times \phi_D(x_2, b_2) \{[-\phi_P^A(x_3)r_D(r_c + x_1 - x_2) + r_P \\ & \times [-\phi_P^T(x_3)(-r_c - x_1 + x_3) + \phi_P^P(x_3)(r_c + x_1 \\ & - x_3)]]\alpha_s(t_g) h_e(\beta_g, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_g)] \\ & + [-\phi_P^A(x_3)r_D(-r_b + x_1 + x_2 - 1) + r_P[(-r_b \\ & + x_1 + x_3 - 1)(\phi_P^P(x_3) + \phi_P^T(x_3))] \alpha_s(t_h) \\ & \times h_e(\beta_h, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_h)]\}, \end{aligned} \quad (48)$$

$$\begin{aligned} \mathcal{M}_a^{SP} = & -\frac{8}{3}C_F f_B \pi M_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_2 db_1 db_2 \\ & \times \phi_D(x_2, b_2) \{[-\phi_P^A(x_3)(x_1 - x_3 - r_c) + r_P r_D \\ & \times [-\phi_P^T(x_3)(x_2 - x_3) + \phi_P^P(x_3)(4r_c - 2x_1 + x_2 \\ & + x_3)]]\alpha_s(t_g) h_e(\beta_g, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_g)] \\ & + [-\phi_P^A(x_3)(r_b + x_1 + x_2 - 1) + r_P r_D[(-4r_b \\ & - 2x_1 - x_2 x_3 + 2)\phi_P^P(x_3) - (x_2 - x_3)\phi_P^T(x_3)]] \\ & \times \alpha_s(t_h) h_e(\beta_h, \alpha_a, b_1, b_2) \exp[-S_{gh}(t_h)]\}, \end{aligned} \quad (49)$$

where  $r_D = m_D/M_B$ ,  $r_b = m_b/M_B$ ,  $C_F = 4/3$  is a color factor,  $r_P = m_P^0/M_B$ , with  $m_P^0$  as the chiral mass of the pseudoscalar meson P.

For the  $D_{(s)}$  meson wave function, we adopt the same model as of the B meson:

$$\phi_{D_{(s)}}(x, b) = N_{D_{(s)}} [x(1-x)]^2 \exp \left( -\frac{x^2 m_{D_{(s)}}^2}{2\omega_{D_{(s)}}^2} - \frac{1}{2}\omega_{D_{(s)}}^2 b^2 \right) \quad (50)$$

with  $\omega_D = 0.6$  GeV.

We show here the functions  $h_e$ , coming from the Fourier transform of hard kernel.

$$\begin{aligned} h_e(\alpha, \beta, b_1, b_2) &= h_1(\alpha, b_1) \times h_2(\beta, b_1, b_2), \\ h_1(\alpha, b_1) &= \begin{cases} K_0(\sqrt{\alpha}b_1), & \alpha > 0 \\ K_0(\sqrt{-\alpha}b_1), & \alpha < 0 \end{cases} \\ h_2(\beta, b_1, b_2) &= \begin{cases} \theta(b_1 - b_2) I_0(\sqrt{\beta}b_2) K_0(\sqrt{\beta}b_1) \\ \theta(b_1 - b_2) J_0(\sqrt{-\beta}b_2) K_0(i\sqrt{-\beta}b_1) \end{cases} \\ &+ \begin{cases} +(b_1 \leftrightarrow b_2), & \beta > 0 \\ +(b_1 \leftrightarrow b_2), & \beta < 0 \end{cases} \end{aligned} \quad (51)$$

Where  $J_0$  is the Bessel function and  $K_0$ ,  $I_0$  are modified Bessel function with  $K_0(ix) = \frac{\pi}{2}(-N_0(x) + iJ_0(x))$ . The hard scale  $t$  is chosen as the maximum of the virtuality of the internal momentum transition in the hard amplitudes, including  $1/b_i$  ( $i = 1, 2, 3$ ):

$$\begin{aligned} t_a &= \max\{\sqrt{|\alpha_e|}, \sqrt{|\beta_a|}, 1/b_1, 1/b_2\}, \\ t_b &= \max\{\sqrt{|\alpha_e|}, \sqrt{|\beta_b|}, 1/b_1, 1/b_2\}, \\ t_c &= \max\{\sqrt{|\alpha_e|}, \sqrt{|\beta_c|}, 1/b_2, 1/b_3\}, \\ t_d &= \max\{\sqrt{|\alpha_e|}, \sqrt{|\beta_d|}, 1/b_2, 1/b_3\}, \\ t_e &= \max\{\sqrt{|\alpha_a|}, \sqrt{|\beta_e|}, 1/b_2, 1/b_3\}, \\ t_f &= \max\{\sqrt{|\alpha_a|}, \sqrt{|\beta_f|}, 1/b_2, 1/b_3\}, \\ t_g &= \max\{\sqrt{|\alpha_a|}, \sqrt{|\beta_g|}, 1/b_1, 1/b_2\}, \\ t_h &= \max\{\sqrt{|\alpha_a|}, \sqrt{|\beta_h|}, 1/b_1, 1/b_2\}, \end{aligned} \quad (52)$$

where

$$\begin{aligned} \alpha_e &= (1 - x_1 - x_2)(x_1 - r_D^2)M_B^2, \\ \alpha_a &= -x_2x_3(1 - r_D^2)M_B^2, \\ \beta_a &= [r_b^2 - x_2(1 - r_D^2)]M_B^2, \\ \beta_b &= -(1 - x_1)(x_1 - r_D^2)M_B^2, \\ \beta_c &= -(1 - x_1 - x_2)[1 - x_1 - x_3(1 - r_D^2)]M_B^2, \\ \beta_d &= (1 - x_1 - x_2)[x_1 - x_3 - r_D^2(1 - x_3)]M_B^2, \\ \beta_e &= [r_c^2 - x_3 - (1 - x_3)r_D^2]M_B^2, \\ \beta_f &= -x_2(1 - r_D^2)M_B^2, \\ \beta_g &= [r_c^2 - (x_1 - x_3(1 - r_D^2))(x_1 - x_2)]M_B^2, \\ \beta_h &= [r_b^2 - (1 - x_1 - x_3 + x_3r_D^2)(1 - x_1 - x_2)]M_B^2, \end{aligned} \quad (53)$$

The  $S_t$  re-sums the threshold logarithms  $\ln^2 x$  appearing in the hard kernels to all orders and it has been parameterized as

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)} [x(1-x)]^c, \quad (54)$$

with  $c = 0.4$ . In the nonfactorizable contributions,  $S_t(x)$  gives a very small numerical effect to the amplitude [34]. Therefore, we drop  $S_t(x)$  in  $F_e^{LL}$ ,  $F_e^{SP}$ ,  $F_a^{LL}$  and  $F_a^{SP}$ .

The Sudakov factors used in the text are defined by

$$\begin{aligned} S_{ab}(t) &= s \left( \frac{M_B}{\sqrt{2}} x_1, b_1 \right) + s \left( \frac{M_B}{\sqrt{2}} x_2, b_2 \right) \\ &+ \frac{5}{3} \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q(\mu) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\mu) \end{aligned} \quad (55)$$

$$\begin{aligned} S_{cd}(t) &= s \left( \frac{M_B}{\sqrt{2}} x_1, b_2 \right) + s \left( \frac{M_B}{\sqrt{2}} x_2, b_2 \right) \\ &+ s \left( \frac{M_B}{\sqrt{2}} x_3, b_3 \right) + s \left( \frac{M_B}{\sqrt{2}} (1 - x_3), b_3 \right) \\ &+ \frac{11}{3} \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\mu) + 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q(\mu) \end{aligned} \quad (56)$$

$$\begin{aligned} S_{ef}(t) &= s \left( \frac{M_B}{\sqrt{2}} x_2, b_2 \right) + s \left( \frac{M_B}{\sqrt{2}} x_3, b_3 \right) \\ &+ s \left( \frac{M_B}{\sqrt{2}} (1 - x_3), b_3 \right) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\mu) \\ &+ 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q(\mu) \end{aligned} \quad (57)$$

$$\begin{aligned} S_{gh}(t) &= s \left( \frac{M_B}{\sqrt{2}} x_1, b_1 \right) + s \left( \frac{M_B}{\sqrt{2}} x_2, b_2 \right) \\ &+ s \left( \frac{M_B}{\sqrt{2}} x_3, b_2 \right) + s \left( \frac{M_B}{\sqrt{2}} (1 - x_3), b_2 \right) \\ &+ \frac{5}{3} \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q(\mu) + 4 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\mu) \end{aligned} \quad (58)$$

$\gamma_q = -\alpha_s/\pi$  is the anomalous dimension of the quark. The explicit form for the function  $s(Q, b)$  is:

$$s(Q, b) = \int_{1/b}^Q \frac{d\mu}{\mu} [\ln(\frac{Q}{\mu}) A(\alpha_s(\mu)) + B(\alpha_s(\mu))], \quad (59)$$

where the anomalous dimensions  $A$  two loops and  $B$  to one loop are

$$\begin{aligned} A &= C_F \frac{\alpha}{\pi} + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3} \beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right) \right] \left( \frac{\alpha_s}{\pi} \right)^2, \\ B &= \frac{2\alpha_s}{3\pi} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right), \end{aligned} \quad (60)$$

with  $C_F = 4/3$  a color factor and  $\gamma_E$  the Euler constant. The one-loop expression of the running coupling constant,

$$\alpha_\mu = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \quad (61)$$



is substituted into Eq.(59) with the coefficient  $\beta_0 = (33 - 2n_f)/3$ .  $n_f$  is the number of the quark flavors.

Here, we specify the light-cone distribution amplitudes(LCDAs) for pseudoscalar and vector mesons. The twist-2 pseudoscalar meson distribution amplitude  $\phi_p^A(P = \pi, K)$ , and the twist-3 ones  $\phi_p^P$  and  $\phi_p^T$  have been parametrized as [33],

$$\phi_p^A(x) = \frac{f_P}{\sqrt{6}} 3x(1-x)[1 + a_1^P C_1^{3/2}(t) + a_2^P C_2^{3/2}(t) + a_4^P C_4^{3/2}(t)] \quad (62)$$

$$\phi_p^P(x) = \frac{f_P}{2\sqrt{6}} [1 + (30\eta_3 - \frac{5}{2}\rho_P^2)C_2^{1/2}(t) - 3(\eta_3\omega_3 + \frac{9}{20}\rho_P^2(1 + 6a_2^P))C_4^{1/2}(t)] \quad (63)$$

$$\phi_p^T(x) = \frac{f_P}{2\sqrt{6}} (1 - 2x)[1 + 6(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_P^2 - \frac{3}{5}\rho_P^2 a_2^P)(1 - 10x + 10x^2)] \quad (64)$$

Where  $t = 2x - 1$ . For pseudoscalar mesons, we choose  $\eta_3 = 0.015$  and  $\omega_3 = -3$ . The mass ratio  $\rho_{\pi(K)} = m_{\pi(K)}/m_0^{\pi(K)}$  and  $\rho_{\eta_q(s)} = 2m_{q(s)}/m_{qq(ss)}$ , and the Gegenbauer polynomials  $C_n^\nu(t)$  read

$$\begin{aligned} C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(+3 - 30t^2 + 35t^4), \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), \quad C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4), \\ C_1^{3/2}(t) &= 3t. \end{aligned} \quad (65)$$

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